Math and Caves

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> A topic of math is like a cave. A long, deep, branching complex of tunnels that stretches into the underground that never presents more than a mere cave hole that serves as its entrance. My objective of exploring math is to catch the beautiful sights in these caves, and I think it's a path that can be (and should be) taken by *anyone*.

In school, teachers serve as tour guides who lead troops of students into pre-selected parts of the cave (syllabus) and point out some parts of the scenery. That scenery often has much hidden beauty, but just as often the pressure of limited curriculum time hastens the pace of teachers and students, leaving them scant time to discover that beauty. "The shape of the stalactites will be in the next test. Oh and don't forget, you guys still owe me the height measurement exercises of last week's Cave Chamber 2.1!" (note: 'Cave Chamber' ↔ 'Notes Chapter') Like busy tourists, the students hurriedly snap a few photos of stalactites and shuffle on to Cave Chamber 2.4. A bored student mutters furtively, "Once I submit my Cave Review, I'm never setting foot in this boring cave again." Onlookers nod in assent.

Of course, this doesn't happen all of the time, but it does often enough, and that scares potential cave adventurers (Mathematicians) away. Thankfully, it's not only tour agencies (schools) that show us part of caves according to their tour packages (syllabi). Many professional cave adventurers write books about their exploits and insert beautiful pictures, like one of an underground pool colored emerald green by minerals, or mysterious rock formations that take the shape of mythological creatures. A friend whose hobby is amateur cave adventuring might relate to you his exciting adventures from just last week, and whip out the proof from his digital camera. These sources of positive influence make one go, "Hey, maybe there *is* something worth checking out in there."

Cave adventuring for the first time is no joke. To avoid taking wrong turns and getting lost, you have to stock up on maps drawn by previous adventurers (previous literature) that show fairly well-established areas of exploration. These maps often come with big red crosses marking places that hold gems of discovery, and it's really fun to walk around those lesser-known "tourist attractions" to admire the scenery. As you get the hang of trudging over uneven terrain and crawling through tight tunnels (growing in reasoning power), you gain confidence and explores caves with greater speed and depth. Eventually, some impulse spurs

one to delve into an uncharted shaft, straying off the documented regions and beginning your own discovery. Many of these random excursions just lead to a dead end, but on some fateful day you push away an obstructing piece of rock to uncover a hidden oasis – an undergound waterfall that feeds a lake, mineral deposits forming shards of green and purple crystal near the banks, and prehistoric blind fish that still roam the waters. Now, that's what you call epiphany. You can't describe that feeling of revelation, the beauty of the truth that sweeps all thought from your head. That will be the source of your cave-adventuring motivation for the rest of your life. That is the moment that transforms the casual tourist into the passionate adventurer.

With further cave-going experience, guidance from mentors and a bit of luck, you'll join the league of extraordinary cave adventurers (community of professional Mathematicians) which is full of people you once felt were out of reach. They march through rough cave tunnels at astounding speed, sidestepping pitfalls and avoiding dead ends (flawless logical reasoning) without so much as breaking a sweat! They refer to chambers of interest with illustrious names such as "Home of the Dragon Rock" or "Ruby Chamber" instead of Chamber 4.12. You hear snatches of their conversation which would have sounded like gibberish to you a few years ago:

"Hey, I heard you visited the Crab Swamp yesterday."

(I heard you were thinking about this Theorem yesterday.)

"Yeah, I tried scaling the cliff behind it, but the handholds are too loose - they crumble away beneath my fingers!"

(I wanted to explore new ground, but I couldn't get any sound argument!)

You see a flash of inspiration and decide to butt in: "I know that cliff - if you go through the Dungeon of Bats, you exit halfway up the cliff - I think I saw some solid outcrops you could climb on."

(If you use this other Theorem, you can get somewhere closer - I think you can find a solid logical foundation there.)

Okay, so now you are a confident cave adventurer who knows the cave's usual haunts inside-out... it's not over ;) You're still inside your cave, be it Linear Algebra, Geometry or Calculus, and each cave has its unique beautiful scenery. Maybe you're more into crystal formations or cave life-forms, so you prefer some caves over others, but exploring many caves (broadening your knowledge) helps you attain new heights in adventuring. One day, as you are crawling around the depths of the Linear Algebra Cave, you catch a glimpse of what looks like an passageway blocked by a piece of rock. "Wait a minute... could that be...!?" Your hands tremble in excitement as you shove the obstruction away and you step into the passageway, finding yourself in the Geometry Cave instead. Stunned, you realize that the two caves are connected, and more adventuring will reveal many, many more links. You start to wonder if the two caves should even be divided, or if they should be the same cave. In fact, you find connections between Geometry and Calculus, Topology and Linear Algebra... You begin to understand that all of the caves are interconnected in a vast, complex matrix of tunnels and chambers, all united in one entity, stretching into unfathomable depths beyond. You can't help but stare in awe at this humongous, intricately structured collection of **Eternal Truth** that we've been trying to fathom for millenia. It was all here long before us, and it will continue to exist long after we're gone.

That is **Mathematics**. Happy cave-adventuring! ˆ‿ˆ